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The Black Hole and Cosmological Solutions in IR modified Hořava Gravity

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Abstract

Recently Hořava proposed a renormalizable gravity theory in four dimensions which reduces to Einstein gravity with a *non-vanishing* cosmological constant in IR but with improved UV behaviors. Here, I study an IR modification which breaks "softly" the detailed balance condition in Hořava model and allows the asymptotically *flat* limit as well. I obtain the black hole and cosmological solutions for "arbitrary" cosmological constant that represent the analogs of the standard Schwarzschild-(A)dS solutions which can be asymptotically (A)dS as well as flat and I discuss their thermodynamical properties. I also obtain solutions for FRW metric with an arbitrary cosmological constant. I study its implication to the dark energy and find that it seems to be consistent with current observational data.

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Recently Hořava proposed a renormalizable gravity theory in four dimensions which reduces to Einstein gravity with a *non-vanishing* cosmological constant in IR but with improved UV behaviors [1, 2]. Since then various aspects and solutions have been studied [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29]. In [7], it has been pointed out that the black hole solution in the Hořava model does not recover the usual Schwarzschild-AdS black hole even though the general relativity is recovered in IR at the action level. (For a generalization to topological black holes, see [11].) On the other hand, in [20] an IR modification which allows the flat Minkowski vacuum has been studied by introducing a term proportional to the Ricci scalar of the three-geometry $\mu^4 R^{(3)}$, while considering "vanishing" cosmological constant ($\sim \Lambda_W$) in the Hořava gravity. (For related discussions, see also [10, 17].)

In this paper, I consider the black hole and cosmological solutions in the generalized model with the IR modification term $\mu^4 R^{(3)}$ but with an "arbitrary" cosmological constant in the Hořava gravity. These solutions represent the analogs of the standard Schwarzschild-(A)dS solutions which have been absent in the original Hořava model. I discuss their thermodynamical properties also.

To this ends, I start by considering the ADM decomposition of the metric

$$ds^2 = -N^2 c^2 dt^2 + g_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right) \tag{1}$$

and the IR-modified Hořava action which reads

$$\begin{aligned} S \ = \ & \int dt d^3x \sqrt{g} N \left[\frac{2}{\kappa^2}\left(K_{ij}K^{ij} - \lambda K^2\right) - \frac{\kappa^2}{2\nu^4} C_{ij}C^{ij} + \frac{\kappa^2 \mu}{2\nu^2} \epsilon^{ijk} R^{(3)}_{i\ell} \nabla_j R^{(3)\ell}{}_k \right. \\ & \left. - \frac{\kappa^2\mu^2}{8} R^{(3)}_{ij} R^{(3)ij} + \frac{\kappa^2\mu^2}{8(3\lambda-1)}\left(\frac{4\lambda-1}{4}(R^{(3)})^2 - \Lambda_W R^{(3)} + 3\Lambda_W^2\right) + \frac{\kappa^2\mu^2\omega}{8(3\lambda-1)} R^{(3)} \right] \end{aligned} \tag{2}$$

where

$$K_{ij} = \frac{1}{2N}\left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i\right) \tag{3}$$

is the extrinsic curvature,

$$C^{ij} = \epsilon^{ik\ell} \nabla_k \left(R^{(3)j}{}_\ell - \frac{1}{4} R^{(3)} \delta^j_\ell \right) \tag{4}$$

is the Cotton tensor, $\kappa, \lambda, \nu, \mu, \Lambda_W$, and ω are constant parameters. The last term, which has been introduced in [2, 10, 20], represents a "soft" violation of the "detailed balance" condition in [2] and this modifies the IR behaviors [1].

Let us consider now a static, spherically symmetric solution with the metric ansatz

$$ds^2 = -N(r)^2 c^2 dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \ . \tag{5}$$

[1] In [20], $\omega = 8\mu^2(3\lambda - 1)/\kappa^2$ has been considered for the AdS case, but ω may be considered as an independent parameter, more generally.

By substituting the metric ansatz into the action (2), the resulting reduced Lagrangian, after angular integration, is given by

$$\mathcal{L} = \frac{\kappa^2\mu^2}{8(1-3\lambda)}\frac{N}{\sqrt{f}}\left[(2\lambda-1)\frac{(f-1)^2}{r^2} - 2\lambda\frac{f-1}{r}f' + \frac{\lambda-1}{2}f'^2 \right. \\ \left. - 2(\omega-\Lambda_W)(1-f-rf') - 3\Lambda_W^2 r^2\right] , \tag{6}$$

where the prime ($'$) denotes the derivative with respect to r. In [20], only the asymptotically Minkowski solution with $\Lambda_W \to 0$ limit was considered. Here, I obtain the general solution with an "arbitrary" Λ_W.

The equations of motions are

$$(2\lambda-1)\frac{(f-1)^2}{r^2} - 2\lambda\frac{f-1}{r}f' + \frac{\lambda-1}{2}f'^2 - 2(\omega-\Lambda_W)(1-f-rf') - 3\Lambda_W^2 r^2 = 0 ,$$
$$\left(\frac{N}{\sqrt{f}}\right)'\left((\lambda-1)f' - 2\lambda\frac{f-1}{r} + 2(\omega-\Lambda_W)r\right) + (\lambda-1)\frac{N}{\sqrt{f}}\left(f'' - \frac{2(f-1)}{r^2}\right) = 0 \tag{7}$$

by varying the functions N and f, respectively.

For the $\lambda = 1$ case, which reduces to the standard Einstein-Hilbert action in the IR limit, I obtain

$$N^2 = f = 1 + (\omega-\Lambda_W)r^2 - \sqrt{r[\omega(\omega-2\Lambda_W)r^3 + \beta]} , \tag{8}$$

where β is an integration constant[2]. It is easy to see that this reduces to Lü, Mei, and Pope (LMP)'s AdS black hole solution in [7] (I consider $N^2 = f$ always, from now on), by identifying $\beta = -\alpha^2/\Lambda_W$,

$$f = 1 - \Lambda_W r^2 - \frac{\alpha}{\sqrt{-\Lambda_W}}\sqrt{r} \tag{9}$$

for $\omega = 0$, Kehagias and Sfetsos's asymptotically flat solution in [20], by identifying $\beta = 4\omega M$,

$$f = 1 + \omega r^2 - \sqrt{r[\omega^2 r^3 + 4\omega M]} \tag{10}$$

for $\Lambda_W = 0$.

For $r \gg [\beta/\omega(\omega-2\Lambda_W)]^{1/3}$ (by considering asymptotically AdS case of $\Lambda_W < 0$ with $\omega > 0$, $\beta > 0$, for the moment) (8) behaves as

$$f = 1 + \frac{\Lambda_W^2}{2\omega}r^2 - \frac{\beta}{2\sqrt{\omega(\omega-2\Lambda_W)}}\frac{1}{r} + \mathcal{O}(r^{-4}). \tag{11}$$

This agrees with the usual Schwarzschild black hole (by adopting the units of $G = c \equiv 1$)

$$f = 1 - \frac{2M}{r} \tag{12}$$

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[2] If one add another IR modification term $\kappa^2\mu^2(8(3\lambda-1))^{-1}\hat{\beta}\Lambda_W^2$ as in [2, 10], the solution becomes $N^2 = f = 1 + (\omega-\Lambda_W)r^2 - \sqrt{r[\{\omega(\omega-2\Lambda_W) + \hat{\beta}\Lambda_W^2/3\}r^3 + \beta]}$. But this can be obtained by redefining the parameters $\Lambda_W \to \sqrt{1-\hat{\beta}/3}\,\Lambda_W,\ \omega \to \omega + (\sqrt{1-\hat{\beta}/3} - 1)\Lambda_W$ in (8).

for $\Lambda_W = 0$ and with $\beta = 4\omega M$, independently of ω. But for $\Lambda_W \neq 0$, there are corrections in the numerical factors due to ω effect: With $\beta = 4\omega M$, (11) can be re-written as

$$f = 1 + \frac{|\Lambda_W|}{2}\left|\frac{\Lambda_W}{\omega}\right| r^2 - \frac{2M}{\sqrt{1+2|\Lambda_W/\omega|}}\frac{1}{r} + \mathcal{O}(r^{-4}) \tag{13}$$

in which the coefficients slightly disagree with those of the standard Schwarzschild-AdS black hole[3], by the factor '$|\Lambda_W/\omega|$',

$$f = 1 + \frac{|\Lambda_W|}{2} r^2 - \frac{2M}{r}. \tag{14}$$

This solution has a curvature singularity with the power of $r^{-3/2}$ at $r = 0$,

$$\begin{aligned} R &= \frac{-15(\omega M)^{1/2}}{2r^{3/2}} + 12(\omega - \Lambda_W) + \mathcal{O}(r^{3/2}), \\ R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} &= \frac{81\omega M}{4r^3} - \frac{30(\omega M)^{1/2}(\omega - \Lambda_W)}{r^{3/2}} + \mathcal{O}(1), \end{aligned} \tag{15}$$

but *no* curvature singularity at $r = \infty$. Note that this singularity is *milder* than that of Einstein gravity which has $R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} \sim r^{-6}$ at $r = 0$.[4]

For asymptotically AdS, i.e., $\Lambda_W < 0$ (with $\omega > 0$), the solution (8) has two horizons generally and the temperature for the outer horizon r_+ is given by[5]

$$T = \frac{3\Lambda_W^2 r_+^4 + 2(\omega - \Lambda_W)r_+^2 - 1}{8\pi r_+(1 + (\omega - \Lambda_W)r_+^2)}. \tag{16}$$

In Fig.1, the temperature T vs. the horizon radius r_+ is plotted and this shows that asymptotically, i.e., for large r_+, the temperature interpolates between the AdS cases (above two curves) and flat (bottom curve). There exists an extremal black hole limit of the vanishing temperature where the inner horizon r_- meets with the outer horizon r_+ at[6]

$$r_+^* = \sqrt{\frac{-(\omega - \Lambda_W) + \sqrt{(\omega - \Lambda_W)^2 + 3\Lambda_W^2}}{3\Lambda_W^2}} \tag{17}$$

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[3] This seems to be a quite generic behavior of the *broken* "detailed balance". See, for example, [7].

[4] I consider the four-dimensional curvature invariants just for a formal reason, i.e., the comparison with those of Einstein gravity. But, the degree of singularity is unchanged even if the three-dimensional curvature invariants are considered only.

[5] Due to the lack of Lorentz invariance in UV, the very meaning of the horizons and Hawking temperature would be changed from the conventional ones. The light cones would differ for different wavelengths and so different particles with different dispersion relations would see different Hawking temperature T_H and entropies, the Hawking spectrum would not be thermal. But from the recovered Lorentz invariance in IR (with $\lambda = 1$), the usual meaning of the horizons and T as the Hawking temperature would be "emerged" for long wavelengths. The calculation and meaning of the temperature should be understood in this context.

[6] For $\Lambda_W \to 0$ limit, (17) becomes $0/0$. But, from (16), one can get easily $r_+^* = 1/\sqrt{2\omega}$ without the ambiguity.

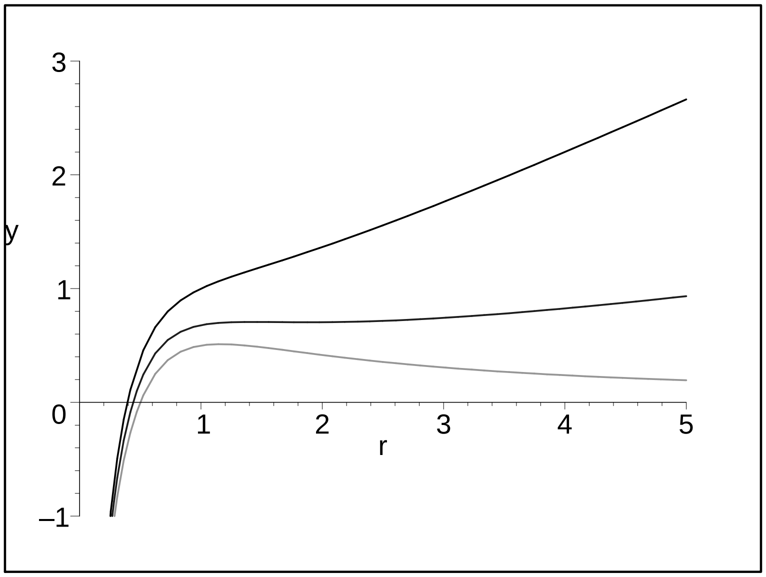

FIG. 1: Plots of $4\pi T$ vs. black hole's outer horizon radius r_+ for various Λ_W, i.e., $\Lambda_W = -1,\ 0.5,\ 0$ (top to bottom) with $\omega = 2$. For large black holes, the temperature interpolates between the AdS cases (above two curves) and flat (bottom curve). In IR region, i.e., small black holes, there exists an extremal black hole limit of the vanishing temperature at r_+^* in which r_- meets with the outer horizon r_+. The region smaller than the extremal radius, which being the Cauchy horizon, shows an unphysical negative temperature.

and the integration constant

$$\beta = \frac{1 + 2(\omega - \Lambda_W) r_+^2 + \Lambda_W^2 r_+^4}{r_+} \tag{18}$$

gets the minimum. The extremal radius r_+^* is the Cauchy horizon and so continuation to region of $r_+ < r_+^*$ does not make sense to outside observer; $T < 0$ for $r_+ < r_+^*$ reflects a pathology of the region. (For some recent related discussions, see [30].)

For asymptotically dS, ie., $\Lambda_W > 0$, the action is given by an analytic continuation[7]

$$\mu \to i\mu,\ \nu^2 \to -i\nu^2,\ \omega \to -\omega \tag{19}$$

of (2) [7]. This can be easily seen in the expansion (13) for $r \gg [\beta/|\omega(\omega - 2\Lambda_W)|]^{1/3}$,

$$f = 1 - \frac{\Lambda_W}{2}\left|\frac{\Lambda_W}{\omega}\right| r^2 - \frac{2M}{\sqrt{1 + 2|\Lambda_W/\omega|}}\frac{1}{r} + \mathcal{O}(r^{-4}) \tag{20}$$

which agrees with the usual Schwarzschild-dS cosmological solution

$$f = 1 - \frac{\Lambda_W}{2} r^2 - \frac{2M}{r}, \tag{21}$$

[7] This corresponds to the analytic continuation of the three-dimensional *Euclidian* action $W_{\rm Euc} = \frac{1}{\nu^2}\int Tr\left(\Gamma \wedge d\Gamma + \frac{2}{3}\Gamma \wedge \Gamma \wedge \Gamma\right) + \mu \int d^3x \sqrt{g}(R^{(3)} - 2\Lambda_W)$ into $iW_{\rm Lor}$ with the real-valued action $W_{\rm Lor}$ [2, 7]. This prescription agrees with [31] but disagrees with [32].

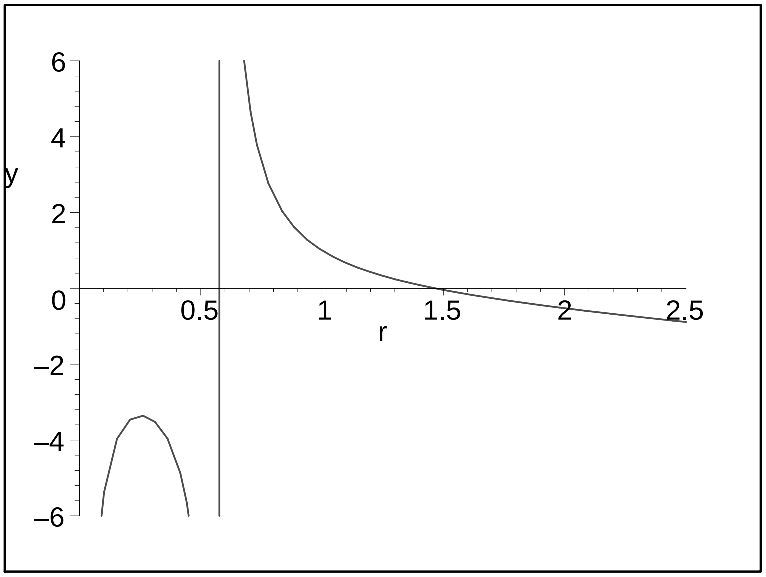

FIG. 2: Plots of $4\pi T$ vs. black hole horizon radius r_+ for the dS black hole case with $\Lambda_W = +1$ and $\omega = -2$. There exists also an extremal limit of the vanishing temperature at r_+^* of (17) in which the black hole horizon r_+ coincides with the cosmological horizon r_{++}, i.e., the *Nariai* limit. There is an infinite discontinuity of temperature at $\tilde{r}_+ = 1/\sqrt{\Lambda_W - \omega}$.

up to some numerical factor corrections. The dS solution also has two horizons generally; the larger one r_{++} for the cosmological horizon and the smaller one r_+ for the black hole horizon.

The black hole temperature is given by (16) also but now with $\Lambda_W > 0, \omega < 0$. There exists also an extremal limit of the vanishing temperature at r_+^* of (17) in which the black hole horizon r_+ coincides with the cosmological horizon r_{++}, i.e., the *Nariai* limit. But a peculiar thing is that there is an infinite discontinuity of temperature at

$$\tilde{r}_+ = \frac{1}{\sqrt{\Lambda_W - \omega}} \tag{22}$$

(see Fig. 2). This may be understood from the following facts. First, by writing $\beta = 4\omega M$ with mass parameter M, in conformity with the usual convention of (21), one needs to consider additionally the condition

$$M \leq \frac{(2\Lambda_W - \omega)}{4} r_+^3 \tag{23}$$

in order that the black hole horizon exists and the curvature singularity at $r = 0$ is not naked. But this inequality is satisfied always from the relation

$$M = \frac{1 + 2(\omega - \Lambda_W) r_+^2 + \Lambda_W^2 r_+^4}{4\omega r_+} : \tag{24}$$

(23) reduces to the condition $[(\omega - \Lambda_W) r_+^2 + 1]^2 \geq 0$, where the equality for $r_+ = 1/\sqrt{\Lambda_W - \omega} = \tilde{r}_+$ corresponds to the upper bound of the mass (23). In other words, $M < \frac{(2\Lambda_W - \omega)}{4} r_+^3$ for all r_+ except for $r_+ = \tilde{r}_+$, where M meets the upper bound

$$M_{\text{bound}} = \frac{(2\Lambda_W - \omega)}{4} r_+^3 = \frac{2\Lambda_W - \omega}{4(\Lambda_W - \omega)^{3/2}}. \tag{25}$$

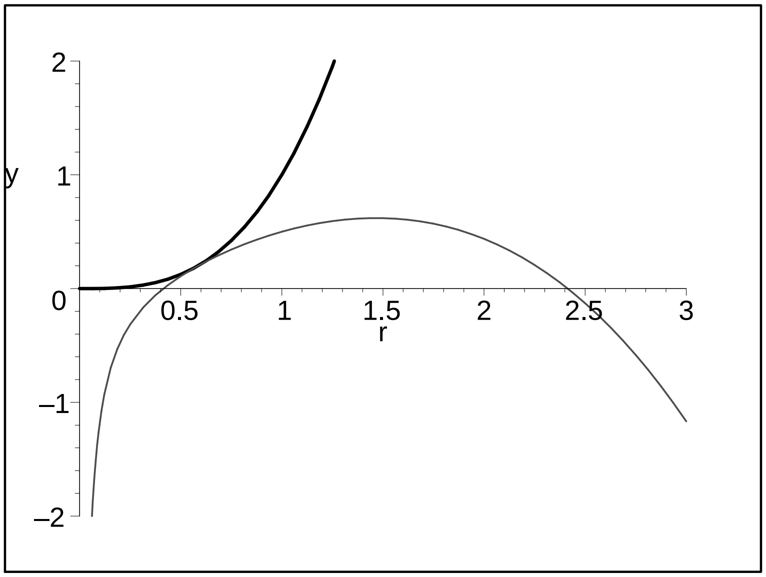

FIG. 3: Plots of mass spectrum M of (24) (lower curve) vs. the upper mass bound $M = \frac{(2\Lambda_W - \omega)}{4} r_+^3$ of (23) (upper curve) for the existence of black hole horizon r_+. This shows that the mass bound is always satisfied for all horizon radius r_+ and this is saturated at the point $r_+ = 1/\sqrt{\Lambda_W - \omega} = \tilde{r}_+$. And also, the mass parameter gets the maximum value in the spectrum at the Nariai limit r_+^*. Here I plotted $\Lambda_W = +1$ and $\omega = -2$ cases but the results are generally valid for arbitrary values of $\Lambda_W > 0$ and $\omega < 0$.

(See Fig.3 for the graphical explanation of this circumstance.) So, the occurrence of the infinite temperature discontinuity and even the negative temperature for $r_+ < \tilde{r}_+$ would be a reflection of being the upper bound of the mass parameter M, for a given r_+, like the spin system with the upper bound of the energy level. (For some recent discussions in different contexts, see also [33].) There is no "geometrical" reason to exclude $r_+ < \tilde{r}_+$, where $T < 0$. But, the negative temperature *might* be a signal of the instability of the smaller black hole, like the negative temperature spin systems in the ordinary surroundings with a positive temperature. This situation is quite different from the asymptotically AdS or flat case, where $T < 0$ region is geometrically protected by the Cauchy horizon at r_+^* in which the inner and outer horizon coincides and $T = 0$.

So far, I have studied the black hole and cosmological solutions for $\lambda = 1$, which matches exactly with the Einstein-Hilbert action in IR. It would be interesting to find the more general solutions for arbitrary values of λ. Especially in cosmology, the arbitrary λ solutions would be also quite important for the practical purpose [20]. So I consider a homogeneous and isotropic cosmological solution to the action (2) with the standard FRW form (by recovering "c")

$$ds^2 = -c^2 dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2/R_0^2} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \right], \qquad (26)$$

where $k = +1, 0, -1$ correspond to a closed, flat, and open universe, respectively, and R_0 is the radius of curvature of the universe in the current epoch. Assuming the matter contribution to be of the form of a perfect fluid with the energy density ρ and pressure p, I

find that

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{6(3\lambda-1)}\left[\rho \pm \frac{3\kappa^2\mu^2}{8(3\lambda-1)}\left(\frac{-k^2}{R_0^4 a^4} + \frac{2k(\Lambda_W - \omega)}{R_0^2 a^2} - \Lambda_W^2\right)\right], \tag{27}$$

$$\frac{\ddot{a}}{a} = \frac{\kappa^2}{6(3\lambda-1)}\left[-\frac{1}{2}(\rho+3p) \pm \frac{3\kappa^2\mu^2}{8(3\lambda-1)}\left(\frac{k^2}{R_0^4 a^4} - \Lambda_W^2\right)\right]. \tag{28}$$

(I have corrected some typos in [20].) Here I have considered the analytic continuation $\mu^2 \to -\mu^2$ for the dS case, i.e., $\Lambda_W > 0$ [7] and the upper (lower) sign denotes the AdS (dS) case. Note that the $1/a^4$ term, which is the contribution from the higher-derivative terms in the action (2), exists only for $k \neq 0$ and become dominant for small $a(t)$, implying that the cosmological solutions of general relativity are recovered at large scales. The first Friedman equation (27) generalizes those of [7] and [20] to the case with an arbitrary cosmological constant and the soft IR modification term in [2, 10, 20]. However, it is interesting to note that there is *no* contribution from the soft IR modification to the second Friedman equation (28) and this is identical to that of [7].

For vacuum solutions with $p = \rho = 0$, I have

$$\left(\frac{\dot{a}}{a}\right)^2 = \mp\frac{\kappa^4\mu^2}{16(3\lambda-1)^2}\left[\left(\frac{k}{R_0^2 a^2} - \Lambda_W\right)^2 + \frac{2k\omega}{R_0^2 a^2}\right]. \tag{29}$$

Here, the role of the ω term is crucial. Without that term, only the constant solution of $a^2 = -1/\Lambda_W R_0^2$ with $k = -1$ exists when $\Lambda_W < 0$, otherwise $(\dot{a}/a)^2$ becomes negative [7]. But now with the last term I have more possibilities: There may exist *non-constant* solutions even for $k = +1$ if $-\omega$ is big enough to make $(\dot{a}/a)^2 > 0$. Actually, for $-\omega > 2|\Lambda_W|,\ k = +1$, there exists a cyclic universe solution

$$a_{\rm AdS}^2(t) = \frac{k(-\omega - |\Lambda_W|)}{R_0^2\Lambda_W^2}\left[1 \pm \sqrt{\frac{-\omega(-\omega - 2|\Lambda_W|)}{(-\omega - |\Lambda_W|)^2}}\left|\sin\left(\frac{\kappa^2\mu\Lambda_{\rm W}}{2(3\lambda-1)}({\rm t}-\gamma)\right)\right|\right] \tag{30}$$

which is oscillating between the inner and outer bouncing scale factors

$$a_{\rm AdS}^{\pm} = \frac{\sqrt{-2k\omega} \pm \sqrt{2k(-\omega - 2|\Lambda_W|)}}{2R_0|\Lambda_W|} \tag{31}$$

and the integration constant γ, depending on the initial conditions. The two bouncing scale factors merge as $-\omega$ becomes smaller and coincide at $a_{\rm AdS}^{\pm} = \sqrt{k/|\Lambda_W|R_0^2}$ when $-\omega = 2|\Lambda_W|$. For $k = -1$, the solution reduces to LMP's constant solution when $\omega = 0$, but there is no solution for other values when $\omega < 0$.

On the other hand, for the dS case, i.e., $\Lambda_W > 0$ and $\omega > 0$, the general solution is given by

$$a_{\rm dS}^2(t) = \frac{2|3\lambda-1|}{\kappa^2|\mu|R_0^2\Lambda_W}{\rm e}^{\pm\frac{\kappa^2|\mu|\Lambda_W}{2|3\lambda-1|}(t-\gamma)} + \frac{k^2\kappa^2|\mu|\omega(\omega - 2\Lambda_W)}{8|3\lambda-1|R_0^2\Lambda_W}{\rm e}^{\mp\frac{\kappa^2|\mu|\Lambda_W}{2|3\lambda-1|}(t-\gamma)} - \frac{k(\omega - \Lambda_W)}{R_0^2\Lambda_W^2}. \tag{32}$$

For $k = -1$, a bounce occurs at

$$a_{\rm dS}^{\pm} = \frac{\sqrt{-2k\omega} \pm \sqrt{-2k(\omega - 2\Lambda_W)}}{2R_0\Lambda_W} \tag{33}$$

when $\omega > 2\Lambda_W$; at $a_{\rm dS}^+$ when $a(t)$ shrinks toward $a_{\rm dS}^+$, at $a_{\rm dS}^-$ when $a(t)$ expands toward $a_{\rm dS}^-$. $\omega = 2\Lambda_W$ is the marginal case where the two bouncing scale factors coincide at $a_{\rm dS}^{\pm} = \sqrt{-k/\Lambda_W R_0^2}$ and the universe evolves monotonically from that point to de Sitter vacuum asymptotically or vice versa. When $\omega < 2\Lambda_W$ (and also for arbitrary values of $\omega > 0$ when $k = +1$), the universe evolves from the big bang singularity to de Sitter vacuum or vice versa. For $\omega = 0,\ k = +1$, this reduces to the LMP's solution with the minimum scale factor $a_{\rm min} = 1/\sqrt{\Lambda_W} R_0$ [7][8].

In addition to the evolution of the universe, there are very strong constraints on the equation of state parameters for the constituents of our universe. So, it would be an important test of our Hořava gravity, whose additional contributions to the Friedman equation may not be distinguishable from those of dark energy, to see if one can meet the correct observational constraints[9]. Then, it is easy to see that the energy density and pressure of the dark energy part are given by (for a related discussion with matters in the context of the original Hořava gravity, see [34])

$$\rho_{\rm D.E.} = \pm\frac{3\kappa^2\mu^2}{8(3\lambda-1)}\left(\frac{-k^2}{R_0^4 a^4} - \frac{2k\omega}{R_0^2 a^2} - \Lambda_W^2\right), \tag{34}$$

$$p_{\rm D.E.} = \mp\frac{\kappa^2\mu^2}{8(3\lambda-1)}\left(\frac{k^2}{R_0^4 a^4} - \frac{2k\omega}{R_0^2 a^2} - 3\Lambda_W^2\right), \tag{35}$$

respectively and the equation of state parameter is given by

$$w_{\rm D.E.} = \frac{p_{\rm D.E.}}{\rho_{\rm D.E.}} = \left(\frac{k^2 - 2k\omega R_0^2 a^2 - 3\Lambda_W^2 R_0^4 a^4}{3k^2 + 6k\omega R_0^2 a^2 + 3\Lambda_W^2 R_0^4 a^4}\right). \tag{36}$$

This interpolates from $w_{\rm D.E.} = 1/3$ in the UV limit to $w_{\rm D.E.} = -1$ in the IR limit but the detailed evolution patten in between them depends on the parameters k, ω, Λ_W (See Fig.4-7). This looks to be consistent with current observational constraints but it seems to be still too early to decide what the right one is [35]. But if I consider the transition point from deceleration phase to acceleration phase, which is given by $a_T = \sqrt{|k|/|\Lambda_W| R_0^2}$ from (28) by neglecting the matter contributions, the formula gives $w_{\rm D.E.} = -1/3$, *independently* of the parameters k, ω, Λ_W. If I use $a_T \sim 1/1.03 \approx 0.9709$ which corresponds to $z_T \sim 0.30$ in the astronomer's parametrization $z = 1/a - 1$, I get $|\Lambda_W| \sim (1.03)^2 R_0^{-2} \approx 1.0609 R_0^{-2}$ for the non-flat universe with $|k| = 1$. And also, if I use $\Omega_k \sim -0.026$ [36] in the current epoch ($a = 1$) for the deviation from the critical density, $\Omega_k = \mu^2 k |\Lambda_W| L_{\rm P}^2 / 2a^2 H^2 R_0^2 M_{\rm P}^2$, Hubble parameter $H \equiv \dot{a}/a$, the ratio of Planck mass and length $M_{\rm P}/L_{\rm P} \equiv 2(3\lambda - 1)/\kappa^2$, $k = -1$, I get $\mu \sim 0.2214 H_0 R_0 M_{\rm P}/L_{\rm P}$ with the current value of Hubble parameter H_0.[10] Finally, if I use $w_{\rm D.E.} \sim -1.08$ in the current epoch, I get $\omega \sim 1.0067 R_0^{-2}$ which predicts the

[8] It is interesting to note that these various scenarios for dS case have been considered earlier by Calcagni in the original Hořava model [3] but it is important to note that these can be "realized" only in our modified model with the ω terms. In Calcagni's notation, the scenarios are categorized by the values of $3c^2\tilde{K}^2 - 4B^2|\tilde{\Lambda}|$ but one can have only $3c^2\tilde{K}^2 - 4B^2|\tilde{\Lambda}| = 0$ if one use the actual values of the parameters for the Hořava model, i.e., (34) and (35). This corresponds to the LMP's solution.

[9] While this paper was being finalized, I met a paper by Mukohyama [28] which propose the dark "matter" as integration constant in Hořava gravity.

[10] I follow the physical convention of Ryden [37] which disagrees with [1, 2].

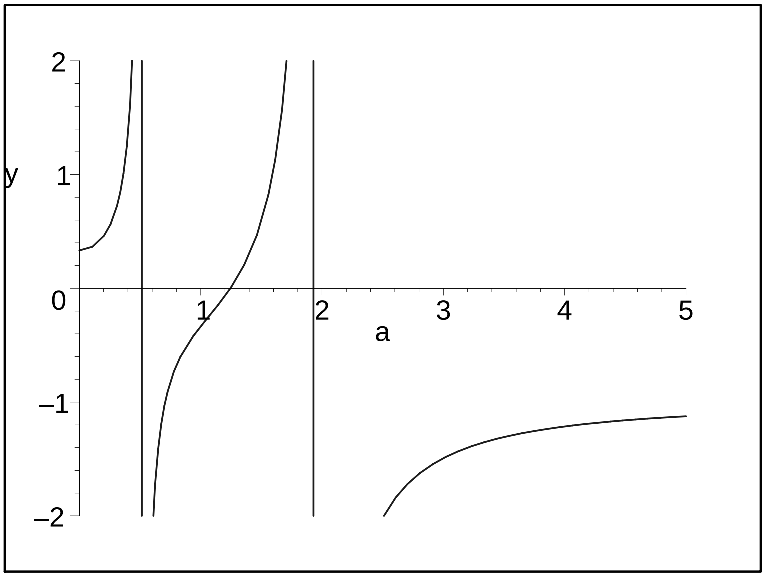

FIG. 4: Plot of equation of state parameter $w_{\rm D.E.}$ vs. scale factor $a(t)$ for $\omega^2 > \Lambda_W^2,\ k\omega < 0$. There are two infinite discontinuities of $w_{\rm D.E.}$ at $\tilde{a}^{\pm} = \sqrt{-k\omega \pm |k|\sqrt{\omega^2 - \Lambda_W^2}}/|\Lambda_W| R_0$ where $\rho_{\rm D.E.}$ vanishes. Here, I considered $|\omega| R_0^2 = 2, |\Lambda_W| R_0^2 = 1$ case ($\omega R_0^2 = -2, k = +1$ or $\omega R_0^2 = +2, k = -1$).

evolution of $w_{\rm D.E.}$ as one of the curves in Fig.6 since $\omega < |\Lambda_W|$: If I use $R_0 \sim 6.2017\ c/H_0$ from $\Omega_k = kc^2/H_0^2 R_0^2 \sim -0.026$ and $H_0 \sim 70$km s^{-1}Mpc^{-1}, I get $\Lambda_W \sim 1.5018 \times 10^{-9}$Mpc^{-2}, $\omega \sim 1.4251 \times 10^{-9}$Mpc^{-2}, $\mu \sim 5.6636 \times 10^{35}$kg s^{-1}.

Note added: After finishing this paper, a related paper [38] appeared whose classification of all the possible cosmology solutions in the Hořava gravity *without* the detailed balance is overlapping with mine. (See also [39].)

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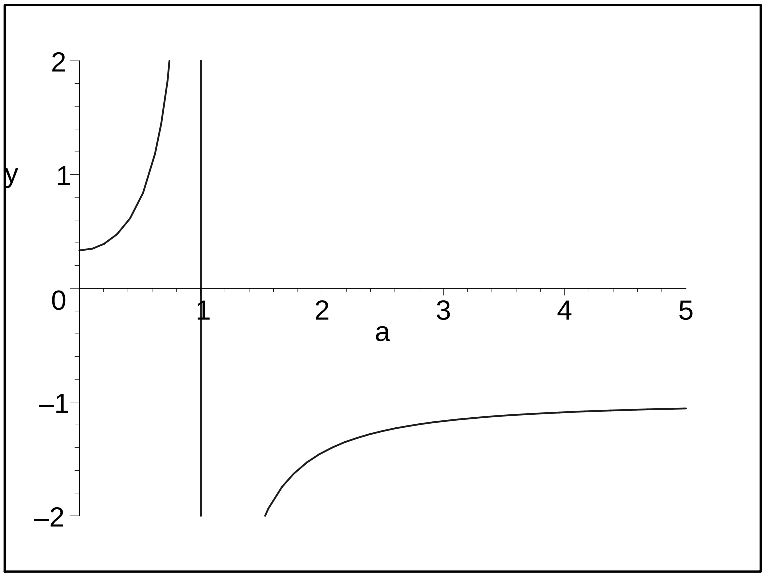

FIG. 5: Plot of equation of state parameter $w_{\text{D.E.}}$ vs. scale factor $a(t)$ for $\omega^2 = \Lambda_W^2,\ k\omega < 0$. The two points of infinite discontinuities $\tilde{a}^{\pm}$ in Fig.4 merge as $|\omega|$ approaches to $|\Lambda_W|$ and they meet at $\tilde{a}^{\pm} = \sqrt{|k|}/|\Lambda_W|R_0$ when $\omega^2 = \Lambda_W^2$. In this plot, I considered $|\omega|R_0^2 = |\Lambda_W|R_0^2 = 1$ ($\omega R_0^2 = -1, k = +1$ or $\omega R_0^2 = +1, k = -1$).

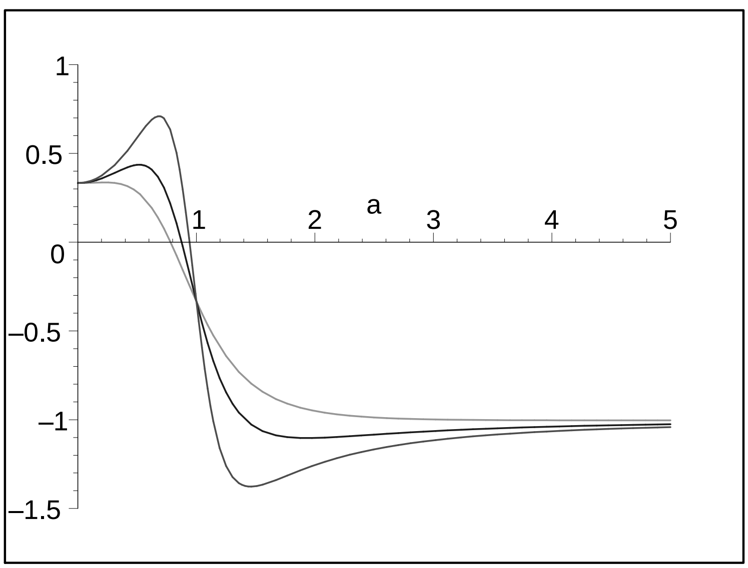

FIG. 6: Plots of equation of state parameters $w_{\text{D.E.}}$ vs. scale factor $a(t)$ for $\omega^2 < \Lambda_W^2,\ k\omega < 0$ ($\omega R_0^2 = +1/1.3, +1/2, +1/10,\ k = -1$ or $\omega R_0^2 = -1/1.3, -1/2, -1/10$, $k = +1$ with $|\Lambda_W|R_0^2 = 1$ (top to bottom in the left region)). When $|\omega|$ is not far from $|\Lambda_W|$, there is a region where $w_{\text{D.E.}}$ is fluctuating beyond the UV and IR limits and this can be understood as a smooth deformation of the plot of Fig.5. When $|\omega|$ is small enough, $w_{\text{D.E.}}$ is monotonically decreasing from 1/3 in the UV limit to -1 in the IR limit.

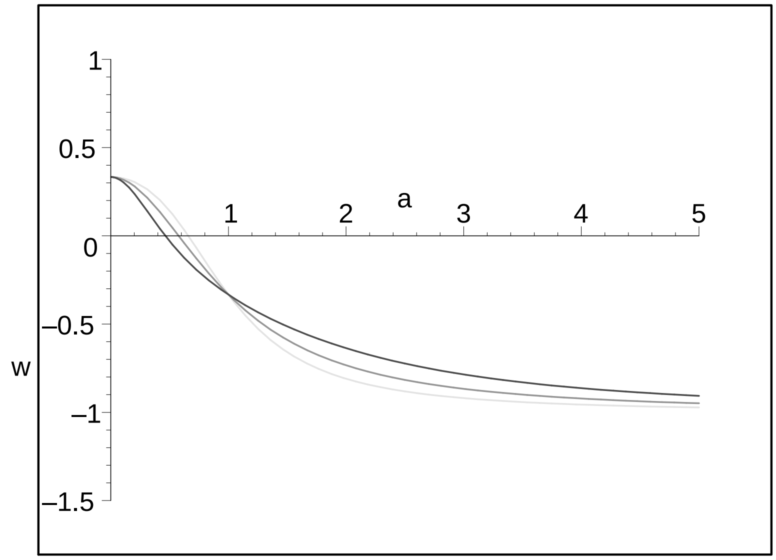

FIG. 7: Plots of equation of state parameters $w_{\rm D.E.}$ vs. scale factor $a(t)$ for $k\omega > 0$ ($\omega R_0^2 = +2, +1, +1/2$, $k = +1$ or $\omega R_0^2 = -2, -1, -1/2$, $k = -1$ with $|\Lambda_W| R_0^2 = 1$ (top to bottom in the left region). In this case, $w_{\rm D.E.}$ is "always" monotonically decreasing from 1/3 in the UV limit to -1 in the IR limit.

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